Engineering Notes

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Simple Model for Predicting Time to Roll Wings Level in the A-7E

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Introduction

IN this Note, we derive and validate a simple model for predicting the time required to roll wings level from an arbitrary bank angle in the A-7E. Although the numerical details of the algorithm are specific to the A-7E, the general approach is applicable to all tactical aircraft.

The simple model of the time to roll wings level described here is used in an improved ground collision avoidance system (GCAS) in the flight software in the A-7E airborne computer. One of the things the GCAS does is calculate the altitude loss incurred in a dive during the time aircraft is rolling to wings level prior to the start of a pull-up. Thus, a model for computing the time to roll wings level from any bank angle and any energy state is needed. Moreover, simplicity is critical since storage and processing time are limited in all airborne computers.

The basic initial-value problem describing the roll behavior of an aircraft is

$$\ddot{\phi} - k\dot{\phi} = F(t), \qquad \phi(0) = \phi_0 \tag{1}$$

where ϕ is the roll angle, k the roll damping coefficient, and $F(t)$ the aileron torque. This initial-value problem is derived, for exmple, in Ref. 1. We assume that $F(t)$ has the "bang-bang" step-response form

$$\begin{aligned} F(t) &= -F_0 \quad \text{for} \quad 0 \le t \le T/2 \\ &= F_0 \quad \text{for} \quad T/2 < t \le T \\ &= 0 \quad \text{for} \quad t > T \end{aligned}$$

where T is the time to roll wings level and F_0 is a constant proportional to the airspeed V. The step response shown was chosen for simplicity. The actual forcing function is complicated due to pilot and aircraft reaction delays. These delays are not included in the calculation of the time to roll wings level, but are included in the A-7E GCAS as fixed times added to the time to roll wings level.

The solution of Eq. (1) is given by

$$\begin{aligned} \phi(t) &= \phi_0 - F_0\,[\exp(-kt) + kt - 1]/k^2 \quad \text{for} \quad 0 \le t < T \\ &= 0 \quad \text{for} \quad t > T \end{aligned}$$

Thus, the time T to roll wings level satisfies the equation

$$0 = \phi_0 - F_0\,[\exp(-kT) + kT - 1)/k^2 \tag{2}$$

Assuming that kT is so large that $\exp(-kT)$ may be neglected safely, we may solve Eq. (2) for T to obtain

$$T = \frac{1}{k} + k\frac{\phi_0}{F_0}$$

Since F is proportional to V, we may write $F_0 = cV$ for a constant c, and, thus,

$$T = \frac{1}{k} + \frac{k}{c}\frac{\phi_0}{V} \tag{3}$$

Equation (3) suggests that T has the form $T = A + B\phi_0/V$. To determine the coefficients A and B, a least-squares linear

Table 1 Flight test and predicted times to roll wings level

Case no.	ϕ_0, deg	V ft/s	T, flight test, s	T, Eq. (4) s	Difference
1	124	317	2.6	2.7	0.1
2	43	350	1.3	1.4	0.1
3	129	353	2.4	2.6	0.2
4	129	373	2.6	2.5	−0.1
5	113	375	2.2	2.3	0.1
6	130	375	2.4	2.5	0.1
7	43	375	1.0	1.3	0.3
8	45	380	1.1	1.3	0.2
9	177	380	2.6	3.0	0.4
10	90	380	1.9	1.9	0.0
11	84	385	1.8	1.8	0.0
12	88	390	2.0	1.9	−0.1
13	87	390	1.8	1.9	0.1
14	175	390	3.0	3.0	0.0
15	170	390	3.1	2.9	−0.2
16	85	395	2.0	1.8	−0.2
17	42	400	1.2	1.3	0.1
18	70	400	1.5	1.6	0.1
19	178	400	2.7	3.0	0.3
20	42	410	1.2	1.3	0.1
21	172	415	2.5	2.8	0.3
22	175	565	2.4	2.3	−0.1
23	42	570	1.0	1.1	0.1
24	43	570	1.1	1.1	0.0
25	126	570	2.0	1.9	−0.1
26	83	580	1.7	1.5	−0.2
27	125	590	2.1	1.8	−0.3
28	88	595	1.7	1.5	−0.2
29	87	600	1.9	1.5	−0.4
30	170	600	2.5	2.2	−0.3
31	44	605	1.0	1.1	0.1
32	174	610	2.6	2.2	−0.4
33	170	615	2.4	2.2	−0.2
34	174	615	2.3	2.2	−0.1
35	41	620	1.1	1.1	0.0
36	169	620	2.6	2.2	−0.4
37	127	630	1.4	1.8	0.4
38	77	635	1.2	1.4	0.2
39	77	825	1.2	1.4	0.2
40	49	850	0.7	1.0	0.3
41	131	850	1.7	1.5	−0.2
42	172	850	1.6	1.8	0.2
43	130	858	1.6	1.5	−0.1
44	127	866	1.8	1.5	−0.3
45	46	875	0.9	1.0	0.1
46	171	890	1.7	1.7	0.0
47	88	890	1.3	1.3	0.0
48	40	900	0.8	1.0	0.2
49	42	900	1.0	1.0	0.0
50	135	915	1.8	1.5	−0.3

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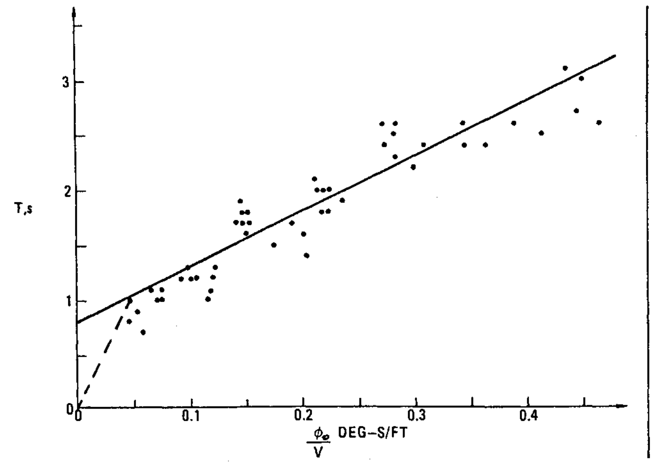

Fig. 1 Plot of flight-test and predicted values of T.

regression of data from flight tests was performed. The data used in this regression are shown in Table 1. Fifty passes using two different aircraft were made to ensure a representative database. Roll angle, speed, and time were obtained from flight data recorders aboard the aircraft. The time T to roll wings level was obtained by subtracting the time at which ϕ began to decrease monotonically from the time at which ϕ was 5 deg or less (thus, pilot and aircraft reaction times are not included in T). The results of the linear regression are $A = 0.8$, $B = 5.0$ when ϕ_0 has units of degrees and V has units of ft/s. Thus, the time T is given approximately by

$$T = 5\phi_0/V + 0.8 \tag{4}$$

The correlation coefficient for the linear fit is 0.93, which indicates a very strong linear correlation.

Values of T obtained from Eq. (4) are shown in Table 1 alongside the corresponding flight-test values. The data in Table 1 indicate that all of the least-squares values of T are within 0.4 s of the flight-test values. However, 90% of the least-squares values of T are within 0.3 s of the corresponding flight-test values. Furthermore, 72% and 50% of the least-squares values of T are within 0.2 s and 0.1 s, respectively, of the corresponding flight-test values. Thus, the data indicate that the linear fit described earlier accurately represents the flight-test data.

Graphical representations of the flight-test data and the line of best fit are shown in Fig. 1. It is again evident that, for most values of ϕ_0/V, the linear fit is accurate. However, for small values of ϕ_0/V, the linear fit appears to be conservative. This is to be expected, since a fundamental assumption made earlier is that the time T to roll wings level is large enough so that the quantity $\exp(-kT)$ may be neglected. For small ϕ_0/V, this condition is clearly not satisfied. A simple solution to this problem is to use a different linear approximation for ϕ_0/V small, say, $\phi_0/V < 0.05$. A reasonable linear equation for this case is the equation that joins (0, 0) and (0.05, 1.0), namely,

$$T = 21\phi_0/V$$

since the flight-test time to roll wings level is about 1.0 s when $\phi_0/V = 0.05$. This equation is plotted as the dashed line in Fig. 1. Thus, the complete algorithm for approximating the time T to roll wings level in the A-7E is

$$\begin{aligned} T &= 5\phi_0/V + 0.8 \quad \text{for} \quad \phi_0/V \geq 0.05 \\ &= 21\phi_0/V \quad \text{for} \quad \phi_0/V < 0.05 \end{aligned}$$

where, again, ϕ_0 is the bank angle in degrees and V is the airspeed in ft/s.

Reference

[1]Perkins, C. D. and Hage, R. E., *Airplane Performance Stability and Control*, Wiley, New York, 1949, pp. 433–436.

Estimation of the Parameters of Convection Dynamics

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Nomenclature

a_0, b_0 = constants, $a_0 \cong 0.12$, $b_0 \cong 0.88$
C_i = fraction of cloud cover of type i
C_p = specific heat at constant pressure for air
E_0 = surface evaporative flux
F_M = wind profile function
F_H = potential temperature profile function
$F_L\downarrow, F_L\uparrow$ = downward and upward long-wave radiative fluxes, respectively
$F_S\downarrow, F_S\uparrow$ = downward and upward short-wave radiative fluxes, respectively
G_0 = energy flux into the ground
g = acceleration due to gravity
H_0 = surface heat flux
h = height of the convective boundary layer
I_0 = solar constant adjusted for variation of orbit radius
k = von Kármán constant, $\cong 0.41$
k_1 = constant, $\cong 0.1$
L = Obukhov length, $L \equiv -u_*^3/(kg/\theta)(H_0/\rho C_p)(1 + 0.61C_pT/\lambda\beta)$
m = absorption of radiation by water vapor, $m \cong 0.18$
P = pressure
R = gas constant for dry air
R_n = net radiation
T, T_g = air temperature, ground temperature
U = horizontal wind speed
u_* = friction velocity, $\equiv (\tau_0/\rho)^{\frac{1}{2}}$
w_* = convective velocity, $\equiv u_*(-h/kL)^{\frac{1}{3}}$
z, z_A, z_s = height, height of anemometer, height of thermometer
z_0, z_H = roughness lengths for momentum and heat
α_1 = constant, $\cong 0.17$
$\alpha_A, \alpha_i, \alpha_g$ = albedo of air, of cloud of type i, and of ground
β = Bowen ratio, $\equiv H_0/\lambda E_0$
γ = ratio of ground flux to net radiation
ε_g = emissivity of the ground, $\varepsilon_g \cong 0.9$
ζ = zenith angle
θ, θ_0 = potential temperature at heights z and z_H
θ_* = temperature scaling parameter, $\equiv -H_0/\rho C_p u_*$
λ = latent heat of vaporization
ξ = dummy variable
ρ = air density
σ = Stefan-Boltzmann constant, $\sigma \cong 5.687 \times 10^{-8}$ $\mathrm{W \cdot m^{-2} \cdot K^{-4}}$
τ_0 = surface stress
ϕ_M, ϕ_H = nondimensional wind shear and potential temperature gradient
ψ_M, ψ_H = buoyancy correction terms to wind and potential temperature profiles

Introduction

THE important role that each of the parameters w_*, h, and $-h/L$ plays in convection dynamics and gust modeling in the boundary layer is described in a companion paper.[1] The

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